Literature Review

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1 Summary

My research is focused on nonlinear conservation laws, an important branch of partial differential equations (PDEs). Nonlinear conservation laws have been used to model a wide array of applications such as fluid dynamics, gas dynamics, and turbulence models. We analyze two applications of nonlinear conservation laws—chemotaxis in biology and traffic flow. Our work in chemotaxis research was published in [35] and parts of our traffic flow research was published in [34] and [33].

2 Chemotaxis

The first problem we investigate concerns a PDE system physically motivated in chemotaxis, which is the movement of organisms due to chemical response. Applications of chemotaxis include self-organization in embryogenesis and the movement of bacteria such as E Coli. The classical mathematical model describing the chemotaxis phenomenon is the Keller-Segel model and was developed in the 1970s [19, 22].

2.1 Overview

We study a specific chemotaxis model that incorporates the concentration of bacteria as well as a function of chemical concentration. It was shown in [32] that the Keller-Segel model can be further modified. In particular, the chemotaxis system we are studying is a special case of the system used in [32]. For this model, we solved the Riemann problem, an initial value problem with piecewise-constant initial condition, which is a building block in the theory of nonlinear conservation laws. One of the main features of this system is that it is of mixed-type in a certain parameter range and changes phases as shown in [27]. In particular, the system has a non-strictly hyperbolic boundary between its elliptic and hyperbolic regions. We were able to overcome this major obstacle of solving the Riemann problem up to the boundary.

2.2 Prior Work in Chemotaxis Research

The Keller-Segel model has been a source of inspiration for mathematical research in chemotaxis and has been studied extensively. Related existing results include existence and stability of traveling waves, global well-posedness and large-time behavior, shock formation, boundary layer formation, etc., see [7, 12, 13, 19, 20, 21, 22, 27, 28, 36, 38, 37, 39, 45, 47, 52]. We are specifically interested in the change of phases. The authors in [27] showed that when the type changes from elliptic to hyperbolic, the solution will collapse to two values, i.e., under certain situations, the solutions will blow up in finite time due to the presence of singularity. In particular, [27] describes scenarios in which we can expect shock formation in chemotaxis models.

2.3 Examples of Similar Phenomenon

The Riemann problem for non-strictly hyperbolic 2×2 systems has been explored in the recent decades by [23, 43, 51]. Various prototypes and examples of non-strictly hyperbolic systems were examined, especially for quadratic models. In these papers, the examples of non-strictly hyperbolic region include isolated umbilic points, axes, or other curves. For example, [56] explores this problem for a genuinely nonlinear system where coinciding characteristic speeds occur on an axis, separating two hyperbolic half-planes. Furthermore, the authors in [18] have explored the Riemann problem for specific mixed-type systems where the non-strictly hyperbolic region divides the u-v plane between the hyperbolic and elliptic regions. Solving Riemann problems for systems is a fundamental topic in conservation laws. Recent work concerned with Riemann problem for non-strictly hyperbolic system has been conducted by LeFloch and De la Cruz among others [11, 16].

2.4 Why is our problem significant?

Similarly, our system also changes phases. In particular, it has a strictly hyperbolic and an elliptic region, with a non-strictly hyperbolic boundary dividing the plane between these two regions. The boundary is a parabola in the right-half plane, which is a distinct feature. This system is genuinely nonlinear in the hyperbolic region, except at $\{u = 0\}$, where it is linearly degenerate. The Riemann problem for non-strictly hyperbolic 2×2 systems has been explored in the recent decades by [23, 43, 51]. In particular, the Riemann problem for a system in chemotaxis was solved by Rascle [49], including in a strictly hyperbolic region in the right half plane and in a linearly degenerate region along $\{u = 0\}$. However, Rascle [49] did not solve the problem up to the non-strictly hyperbolic boundary since the boundary was not in his physically relevant region. In addition, Hillen and Wang described shock wave solutions in parameterized forms for this system [53]. We solved the Riemann problem associated with this chemotaxis model up to the non-strictly hyperbolic boundary as well as on the linearly degenerate region for $u \geq 0$. Our work is significant because we solved a problem that was both mathematically interesting and physically relevant.

3 Traffic Flow

3.1 Overview

For our second problem, we studied a PDE system that models traffic flow. A traffic flow system is of interest because it has physical application based on real-life scenarios. Traffic flow incorporates interactions between roadways, drivers, and vehicles as well as other factors like nonlinear dynamics and human behavior.

3.2 History and Criticism

Several traffic models like the Lighthill-Whitham-Richards (LWR) model [41, 50], Payne-Whitham (PW) model [46, 54], and Aw-Rascle and Zhang's higher continuum models [55] (ARZ) have been studied. The LWR model is the classic model in this field of study and instigated traffic flow research in the mid-1950s with the seminal work, 'Shock Waves on the Highway' [50]. The continuum model offered by LWR was restricted to equilibrium states, while PW incorporated nonequilibrium behavior in their models. It should be noted that [10] criticized the development of traffic flow model in the 1970s and questioned the usefulness of second order models. However, the ARZ equations 'resurrected' the work of second order models by introducing a new model.

3.3 Why is our problem significant?

The model we study is a modified version of the ARZ model, which incorporates an 'anticipation factor' based on the pressure induced by other cars. When the state is in equilibrium, our model reduces to the LWR model. For this system of balance laws arising from traffic flow, we were able to show global existence of bounded variation (BV) solutions to the Cauchy problem under the framework of Dafermos [9]. In particular, we worked with non-concave fundamental diagrams, which have been observed in experimental data from real-life traffic flow [17, 24]. Our research is significant because this is an open problem in nonlinear conservation laws.

3.4 Prior Work in Traffic Flow

Our goal is to verify conditions and show existence of global BV solutions within the context of Dafermos' framework [8, 9] that are needed to find admissible BV solutions to the Cauchy problem. Previously, constructing global BV solutions have been studied in [3, 4, 5, 6, 8, 42, 44]. Microscopic [14], mesoscopic [48] and macroscopic models [1, 2, 6, 15, 24, 25, 26, 30, 40, 31, 41, 46, 50, 55] have been utilized to deal with this phenomenon. Constructing global solutions and finding zero relaxation limits of traffic flow models have been a recent focus of study [15, 25, 26, 30, 29, 31].

In our model we deal with a $g(\rho)$ term, which is a pseudo-pressure function accounting for drivers' anticipation of downstream density changes. While ARZ model adopted a relative wave propagating speed to the car speed at equilibrium [30], we adopt a larger relative speed. Larger relative speed implies quicker reaction time, which leads to safer and smoother traffic conditions on highways.

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